Modular arithmetic

Secondary Mathematics Masterclass

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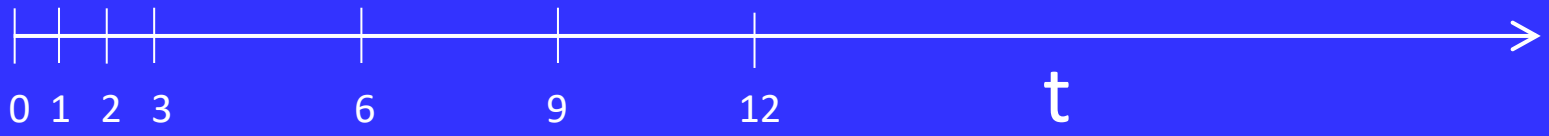
Introduction

On what day were you born?

Worksheet 1
Going round in circles

Modulo 12

How to represent time?



Modulo 12

Instead of 13 = 1, in modular arithmetic we write 13 ≡ 1 (mod 12) and read it “13 is congruent to 1 modulo 12” or, to abbreviate, “13 is 1 modulo 12”.

Examples: $12 \equiv 0 \pmod{12}$ $17 \equiv 5 \pmod{12}$

$37 \equiv 1 \pmod{12}$ $-1 \equiv 11 \pmod{12}$

In general, $a \equiv b \pmod{n}$ if $a-b$ is a multiple of n. Equivalently, $a \equiv b \pmod{n}$ if a and b have the same remainder when divided by n (remainder modulo n).

Clock addition table

+	1	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12	1
2	3	4	5	6	7	8	9	10	11	12	1	2
3	4	5	6	7	8	9	10	11	12	1	2	3
4	5	6	7	8	9	10	11	12	1	2	3	4
5	6	7	8	9	10	11	12	1	2	3	4	5
6	7	8	9	10	11	12	1	2	3	4	5	6
7	8	9	10	11	12	1	2	3	4	5	6	7
8	9	10	11	12	1	2	3	4	5	6	7	8
9	10	11	12	1	2	3	4	5	6	7	8	9
10	11	12	1	2	3	4	5	6	7	8	9	10
11	12	1	2	3	4	5	6	7	8	9	10	11
12	1	2	3	4	5	6	7	8	9	10	11	12

Modulo 12

In modular arithmetic we use the numbers 0-11 instead of the numbers 1-12. The reason is that 0-11 are the remainders modulo 12.

In general, when we work modulo n we replace all the numbers by their remainders modulo n.

Modulo 12 addition table

+	0	1	2	3	4	5	6	7	8	9	10	11
0	0	1	2	3	4	5	6	7	8	9	10	11
1	1	2	3	4	5	6	7	8	9	10	11	0
2	2	3	4	5	6	7	8	9	10	11	0	1
3	3	4	5	6	7	8	9	10	11	0	1	2
4	4	5	6	7	8	9	10	11	0	1	2	3
5	5	6	7	8	9	10	11	0	1	2	3	4
6	6	7	8	9	10	11	0	1	2	3	4	5
7	7	8	9	10	11	0	1	2	3	4	5	6
8	8	9	10	11	0	1	2	3	4	5	6	7
9	9	10	11	0	1	2	3	4	5	6	7	8
10	10	11	0	1	2	3	4	5	6	7	8	9
11	11	0	1	2	3	4	5	6	7	8	9	10

Examples:

$7 + 8 \equiv 3 \pmod{12}$

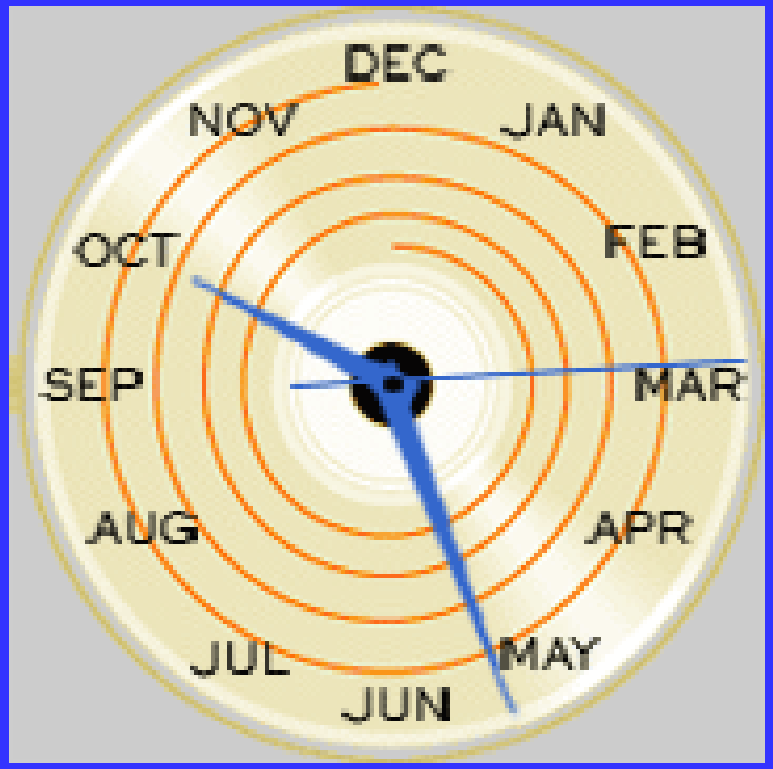
$13 + 2 \equiv 3 \pmod{12}$

$10 + 2 \equiv 0 \pmod{12}$

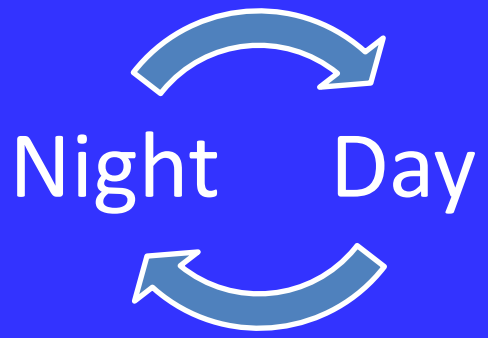
$-1 + 14 \equiv 1 \pmod{12}$

Modulo 12

Can we use arithmetic modulo 12 to represent something else?



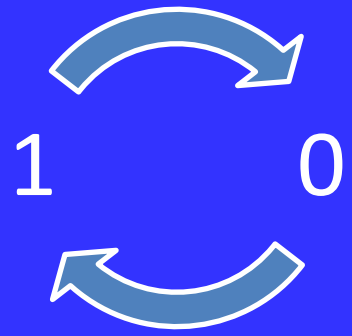
Modulo 2



- We can use 0 to represent Day and 1 to represent Night

Clock with just two numbers

Modulo 2



- 0 and 1 are the remainders modulo 2
- 0 represents the even numbers: 0, 2, 4, 6,…
- 1 represents the odd numbers: 1, 3, 5, 7,…

Algebraically?

2n, any integer n

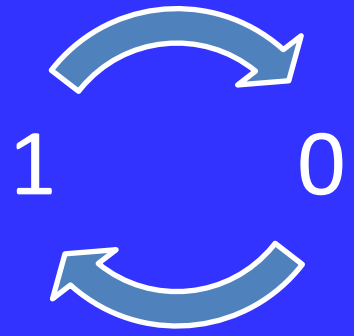
2n+1, any integer n

Examples:

$4 \equiv 0 \pmod 2$ $\quad -6 \equiv 0 \pmod 2$

$13 \equiv 1 \pmod 2$ $\quad -1 \equiv 1 \pmod 2$

Modulo 2 addition table



+	0	1
0	0	1
1	1	0

+	Even	Odd
Even	Even	Odd
Odd	Odd	Even

Examples:

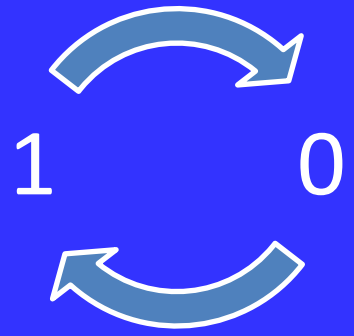
$0 + 1 \equiv 1 \pmod 2$

$13 + 2 \equiv 1 \pmod 2$

$1 + 1 \equiv 0 \pmod 2$

$-1 + 14 \equiv 1 \pmod 2$

Modulo 2 multiplication table



x	0	1
0	0	0
1	0	1

x	Even	Odd
Even	Even	Even
Odd	Even	Odd

Examples:

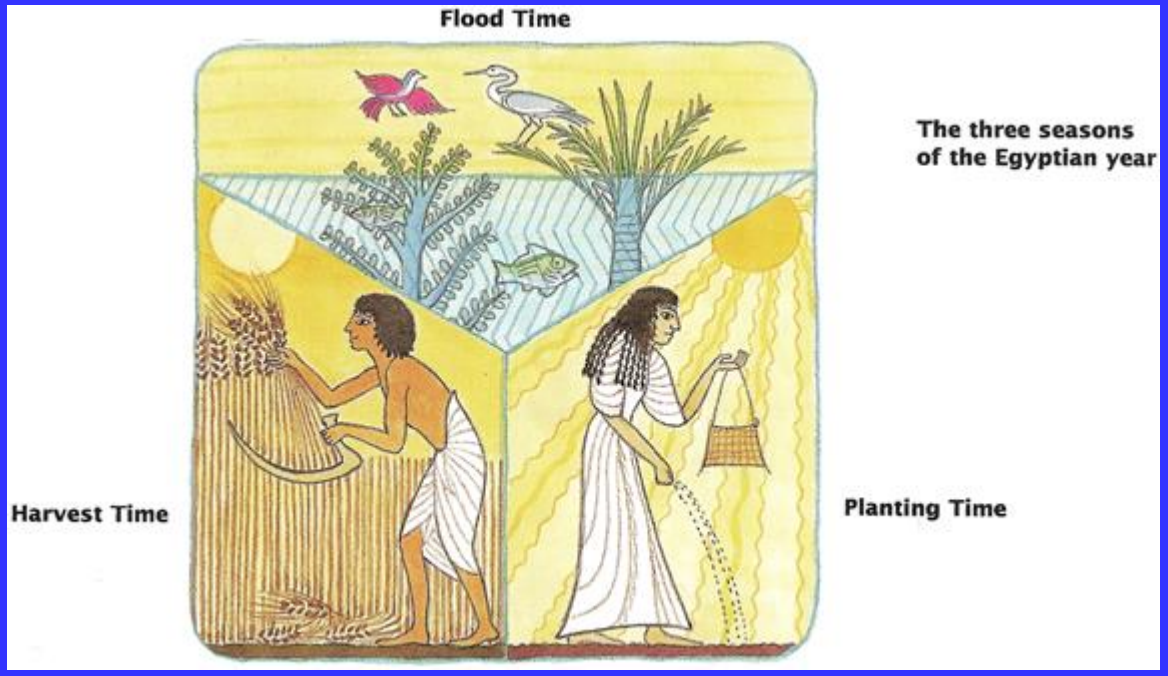
$0 \times 1 \equiv 0 \pmod 2$

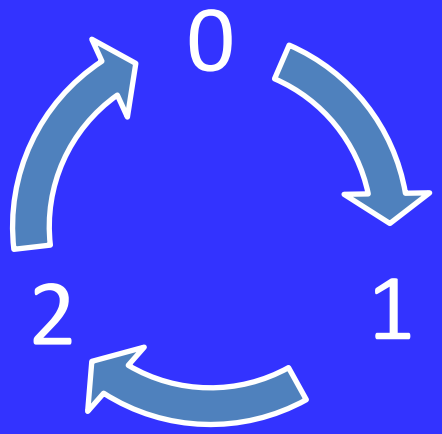
$13 \times 3 \equiv 1 \pmod 2$

$1 \times 1 \equiv 1 \pmod 2$

$-1 \times 14 \equiv 0 \pmod 2$

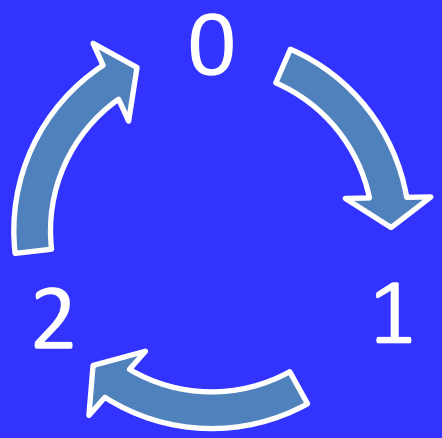
Modulo 3





- 0 represents Flood Time
- 1 represents Planting Time
- 2 represents Harvest Time

Modulo 3



+	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

- 0, 1 and 2 are the remainders modulo 3
- 0 represents the multiples of 3: 0, 3, 6,…
- 1 represents the (multiples of 3) + 1: 1, 4, 7,…
- 2 represents the (multiples of 3) + 2: 2, 5, 8,…

Algebraically?

3n, any integer n

3n+1, any integer n

3n+2, any integer n

Examples:

$3 \equiv 0 \pmod 3$ $\quad -2 \equiv 1 \pmod 3$ $\quad 13 \equiv 1 \pmod 3$

$2 + 2 \equiv 1 \pmod 3$ $\quad -1 + 8 \equiv 1 \pmod 3$ $\quad -2 + 7 \equiv 2 \pmod 3$

Modulo 3 multiplication table

Blackboard

Modulo 3 multiplication table

X	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Remainders

Working modulo n is like wearing special glasses that convert each number into its remainder modulo n. For example, to compute the following sum modulo 12 (to find the remainder modulo 12):

19 + 23 + 15

First replace each number by its remainder mod 12:

7 + 11 + 3

then do the sum: 21

and replace the sum by its remainder modulo 12: 9

Remainders

If today is Sunday, what day will it be in 1000 days?

We need to find the remainder of 1000 when divided by 7.

As we don't need the quotient we don't need to do the division. We look for multiples of 7 lower than 1000:

$1000 = 700+300 = 700+280+20 = 700+280+14+6$

In 6 days, and in 1000 days, it will be Saturday.

Worksheet 2
Remainders and congruences

Remember:

$a \equiv b \pmod{n}$ if $a-b$ is a multiple of n.

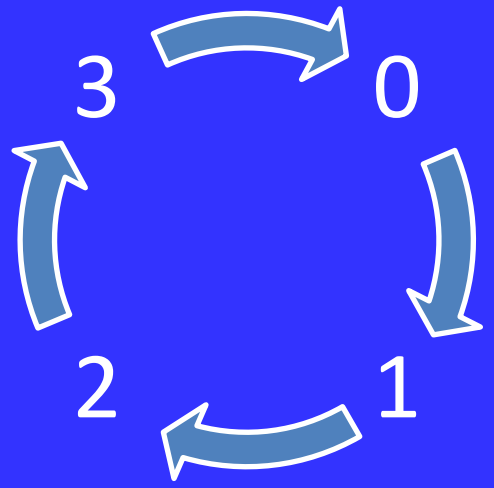
Equivalently,

$a \equiv b \pmod{n}$ if a and b have the same remainder modulo n.

When we work modulo n we replace all the numbers by their remainders modulo n: $0, 1, 2, \ldots, n-1$.

Modulo 4

What can we represent with modulo 4?



- 0 represents Spring
- 1 represents Summer
- 2 represents Autumn
- 3 represents Winter

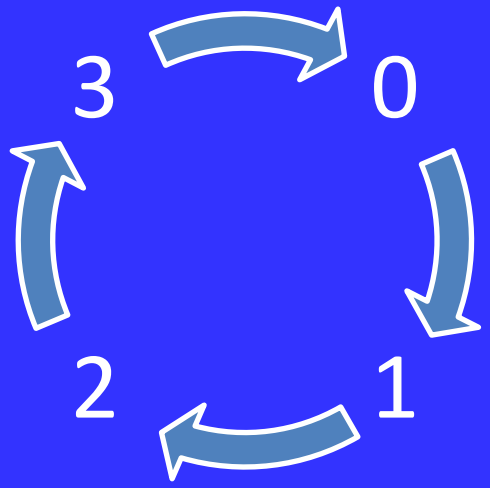
Worksheet 3
Addition and multiplication tables

Remember:

When we work modulo n we replace all the numbers by their remainders modulo n:

$$0, 1, 2, \ldots, n-1$$

Modulo 4 addition table



+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

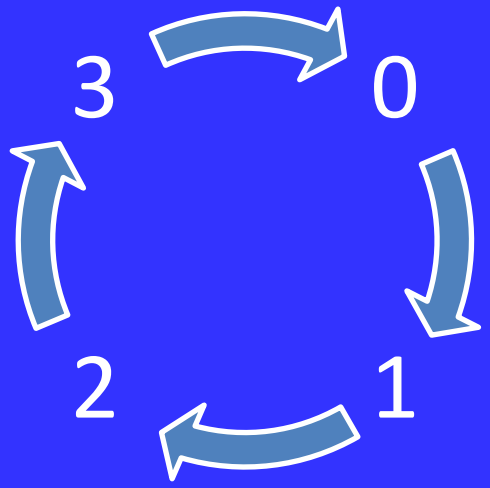
- 0 denotes 4n: 0, 4, 8, 12,…
- 1 denotes 4n+1: 1, 5, 9, 13,…
- 2 denotes 4n+2: 2, 6, 10, 14,…
- 3 denotes 4n+3: 3, 7, 11, 15,…

Examples:

$4 \equiv 0 \pmod 4$ $\quad -2 \equiv 2 \pmod 4$ $\quad 13 \equiv 1 \pmod 4$

$3 + 2 \equiv 1 \pmod 4$ $\quad -1 + 8 \equiv 3 \pmod 4$ $\quad -2 + 7 \equiv 1 \pmod 4$

Modulo 4 multiplication table



x	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

- 0 denotes 4n: 0, 4, 8, 12,…
- 1 denotes 4n+1: 1, 5, 9, 13,…
- 2 denotes 4n+2: 2, 6, 10, 14,…
- 3 denotes 4n+3: 3, 7, 11, 15,…

Examples:

$4 \equiv 0 \pmod 4$ $\quad -2 \equiv 2 \pmod 4$ $\quad 13 \equiv 1 \pmod 4$

$2 \times 2 \equiv 0 \pmod 4$ $\quad 3 \times 2 \equiv 2 \pmod 4$ $\quad 3 \times 3 \equiv 1 \pmod 4$

Last digit arithmetic

What is the last digit of 285714 + 571428?

- It is enough to look at the last digits: 4 + 8 = 12
- Then look at the last digit of their sum: 2

What is the last digit of 142857 x 34745?

- It is enough to look at the last digits: 7 x 5 = 35
- Now look at the last digit of their product: 5

How is this related to modular arithmetic?

Modulo 10

French Revolution clock

- 0 denotes 10n: 0, 10, 20, 30,…
- 1 denotes 10n+1: 1, 11, 21, 31,…
- 2 denotes 10n+2: 2, 12, 22, 32,…
- 3 denotes 10n+3: 3, 13, 23, 33,…
- 4 denotes 10n+4: 4, 14, 24, 34,…
- 5 denotes 10n+5: 5, 15, 25, 35,…
- 6 denotes 10n+6: 6, 16, 26, 36,…
- 7 denotes 10n+7: 7, 17, 27, 37,…
- 8 denotes 10n+8: 8, 18, 28, 38,…
- 9 denotes 10n+9: 9, 19, 29, 39,…

In general: $N \equiv$ last digit of N (mod 10)

Modulo 10 addition table

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	0
2	2	3	4	5	6	7	8	9	0	1
3	3	4	5	6	7	8	9	0	1	2
4	4	5	6	7	8	9	0	1	2	3
5	5	6	7	8	9	0	1	2	3	4
6	6	7	8	9	0	1	2	3	4	5
7	7	8	9	0	1	2	3	4	5	6
8	8	9	0	1	2	3	4	5	6	7
9	9	0	1	2	3	4	5	6	7	8

Examples:

$7 + 4 \equiv 1 \pmod{10}$ $19 + 28 \equiv 7 \pmod{10}$ $-2 + 6 \equiv 4 \pmod{10}$

Modulo 10 multiplication table

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	0	2	4	6	8
3	0	3	6	9	2	5	8	1	4	7
4	0	4	8	2	6	0	4	8	2	6
5	0	5	0	5	0	5	0	5	0	5
6	0	6	2	8	4	0	6	2	8	4
7	0	7	4	1	8	5	2	9	6	3
8	0	8	6	4	2	0	8	6	4	2
9	0	9	8	7	6	5	4	3	2	1

Examples:

$7 \times 4 \equiv 8 \pmod{10}$ $19 \times 28 \equiv 2 \pmod{10}$ $-2 \times 6 \equiv 8 \pmod{10}$

Divisibility by 9

We know that

$$10 \equiv 1 \pmod 9$$

Then $10^2 \equiv 1 \pmod 9$, $10^3 \equiv 1 \pmod 9$, etc.

In general:

$$10^n \equiv 1 \pmod 9 \text{ for any } n$$

Take any number, say 8794, then we have:

$$8794 = 8\text{x}1000 + 7\text{x}100 + 9\text{x}10 + 4$$

$$\equiv 8 + 7 + 9 + 4 \pmod 9$$

Divisibility by 9

- In general we have:

  $N \equiv$ sum of digits of N (mod 9)

- In particular, N is divisible by 9 if and only if the sum of its digits is divisible by 9.

- Given that $10 \equiv 1 \pmod 3$ the same argument proves that $N \equiv$ sum of digits of N (mod 3).

Divisibility by 11

We know that

$10 \equiv -1 \pmod{11}$

Then $10^2 \equiv 1 \pmod{11}$, $10^3 \equiv -1 \pmod{11}$, etc.

In general:

$10^n \equiv 1 \pmod{11}$ if n is even

$10^n \equiv -1 \pmod{11}$ if n is odd

Take any number, say 38,794, then we have:

$38{,}794 = 3\text{x}10{,}000 + 8\text{x}1{,}000 + 7\text{x}100 + 9\text{x}10 + 4$

$\equiv 3 - 8 + 7 - 9 + 4 \pmod{11}$

Divisibility by 11

- In general we have:

  $N \equiv$ alternate sum of digits of N (mod 11)

- In particular, N is divisible by 11 if and only if the alternate sum of its digits is divisible by 11. Start from the right making the units digit positive.

Worksheet 4
Divisibility

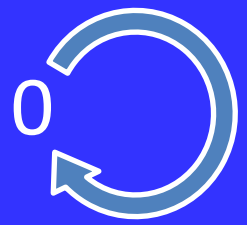
Remember:

- N is divisible by 9 if and only if the sum of its digits is divisible by 9.
- N is divisible by 3 if and only if the sum of its digits is divisible by 3.
- N is divisible by 11 if and only if the alternate sum of its digits is divisible by 11. Start from the right making the units digit positive.

Powers

What is the last digit of:

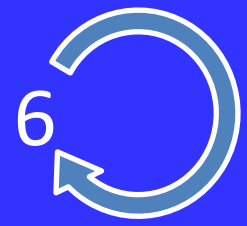
a) 310^{56}



b) 11^{550}

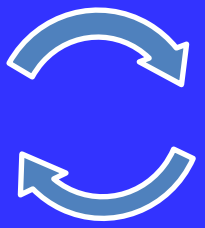
c) $45^{3687682346878922211555657}$

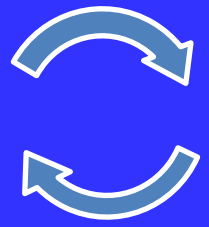
d) 7185787346^{3586}



Powers

What is the last digit of:

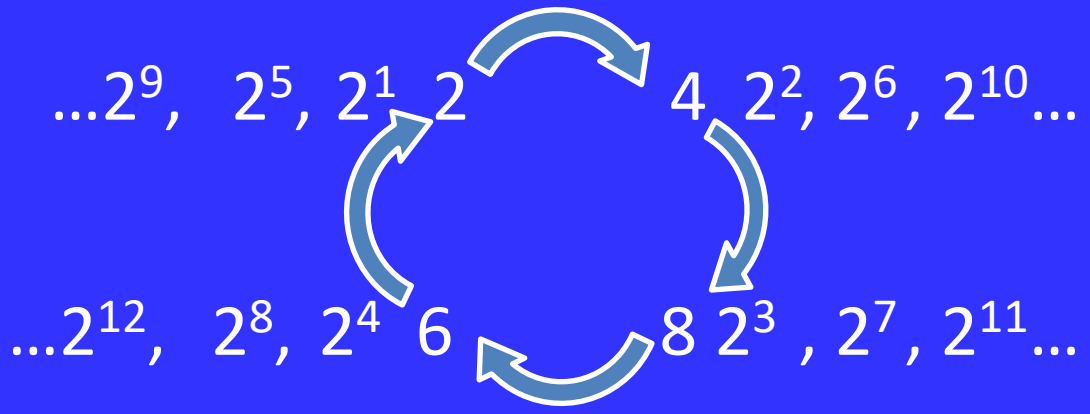
a) 4^{13} …4^5, 4^3, 4^1 4  6 4^2, 4^4, 4^6…

b) 9^{58} …9^5, 9^3, 9^1 9  1 9^2, 9^4, 9^6…

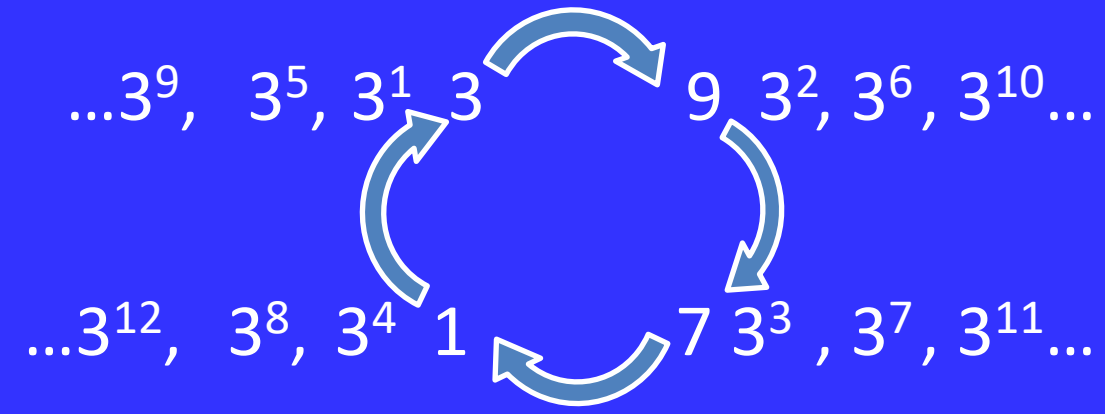
Powers

What is the last digit of:

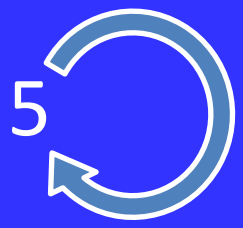
a) 2^{13}



b) 3^{58}



Worksheet 5
Powers



$\ldots 9^5,\ 9^3,\ 9^1\ 9\ 1\ 9^2, 9^4, 9^6\ldots$

$\ldots 2^9,\ 2^5, 2^1\ 2\ 4\ 2^2, 2^6, 2^{10}\ldots$

$\ldots 2^{12},\ 2^8, 2^4\ 6\ 8\ 2^3, 2^7, 2^{11}\ldots$

Modulo 10 multiplication table

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	0	2	4	6	8
3	0	3	6	9	2	5	8	1	4	7
4	0	4	8	2	6	0	4	8	2	6
5	0	5	0	5	0	5	0	5	0	5
6	0	6	2	8	4	0	6	2	8	4
7	0	7	4	1	8	5	2	9	6	3
8	0	8	6	4	2	0	8	6	4	2
9	0	9	8	7	6	5	4	3	2	1

Which of the following are square numbers? 6312, 4553, 9538

Where are the square numbers in the table?

Modulo 10 multiplication table

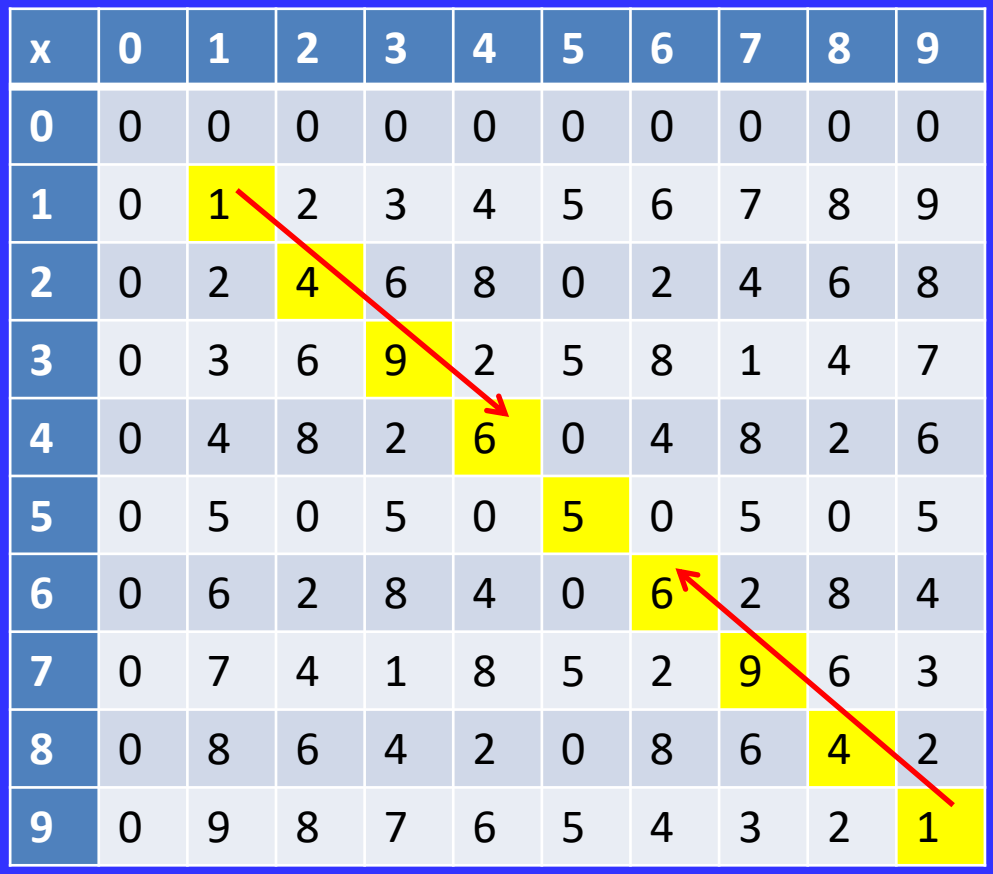
x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	0	2	4	6	8
3	0	3	6	9	2	5	8	1	4	7
4	0	4	8	2	6	0	4	8	2	6
5	0	5	0	5	0	5	0	5	0	5
6	0	6	2	8	4	0	6	2	8	4
7	0	7	4	1	8	5	2	9	6	3
8	0	8	6	4	2	0	8	6	4	2
9	0	9	8	7	6	5	4	3	2	1

Which of the following are square numbers? 6312, 4553, 9538

Where are the square numbers in the table?

Modulo 10 multiplication table

x	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	0	2	4	6	8
3	0	3	6	9	2	5	8	1	4	7
4	0	4	8	2	6	0	4	8	2	6
5	0	5	0	5	0	5	0	5	0	5
6	0	6	2	8	4	0	6	2	8	4
7	0	7	4	1	8	5	2	9	6	3
8	0	8	6	4	2	0	8	6	4	2
9	0	9	8	7	6	5	4	3	2	1

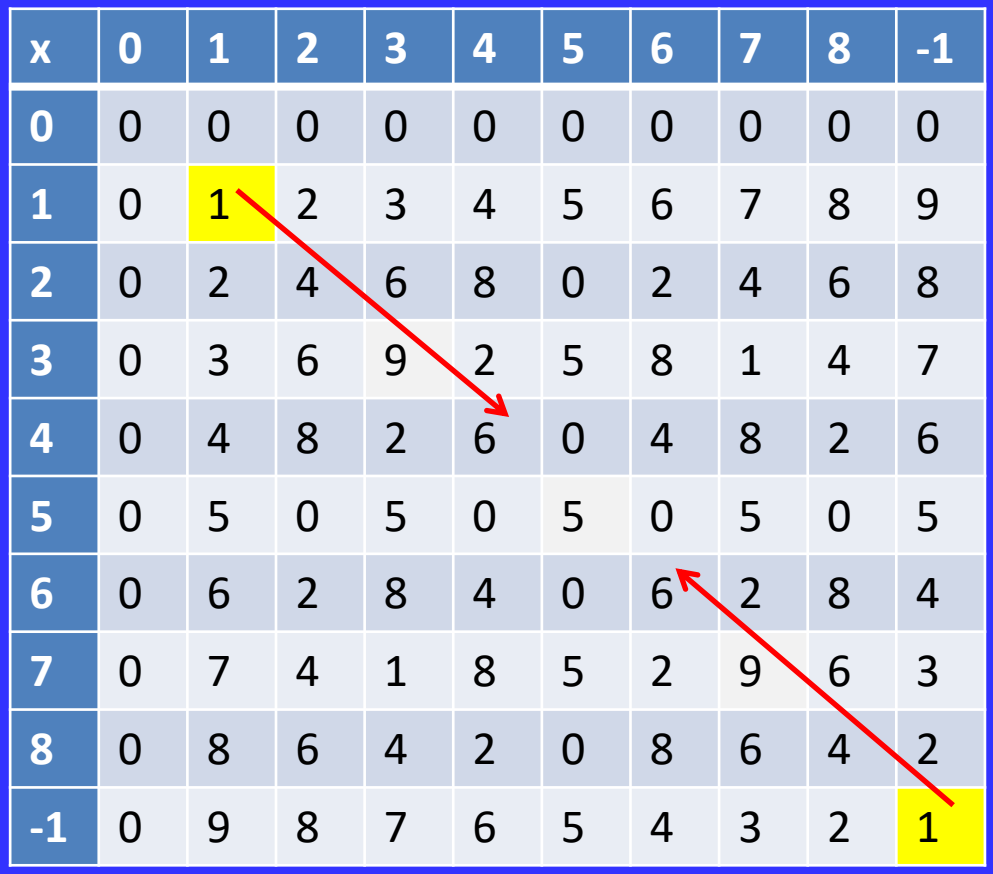


Where are the square numbers in the table?

Why this symmetry?

Modulo 10 multiplication table

x	0	1	2	3	4	5	6	7	8	-1
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	0	2	4	6	8
3	0	3	6	9	2	5	8	1	4	7
4	0	4	8	2	6	0	4	8	2	6
5	0	5	0	5	0	5	0	5	0	5
6	0	6	2	8	4	0	6	2	8	4
7	0	7	4	1	8	5	2	9	6	3
8	0	8	6	4	2	0	8	6	4	2
-1	0	9	8	7	6	5	4	3	2	1



Where are the square numbers in the table?

Why this symmetry? $1^2=(-1)^2$

Modulo 10 multiplication table

x	0	1	2	3	4	5	6	7	-2	-1
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	0	2	4	6	8
3	0	3	6	9	2	5	8	1	4	7
4	0	4	8	2	6	0	4	8	2	6
5	0	5	0	5	0	5	0	5	0	5
6	0	6	2	8	4	0	6	2	8	4
7	0	7	4	1	8	5	2	9	6	3
-2	0	8	6	4	2	0	8	6	4	2
-1	0	9	8	7	6	5	4	3	2	1

Where are the square numbers in the table?

Why this symmetry? $1^2=(-1)^2$, $2^2=(-2)^2$

Modulo 10 multiplication table

x	0	1	2	3	4	5	6	-3	-2	-1
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	0	2	4	6	8
3	0	3	6	9	2	5	8	1	4	7
4	0	4	8	2	6	0	4	8	2	6
5	0	5	0	5	0	5	0	5	0	5
6	0	6	2	8	4	0	6	2	8	4
-3	0	7	4	1	8	5	2	9	6	3
-2	0	8	6	4	2	0	8	6	4	2
-1	0	9	8	7	6	5	4	3	2	1

Where are the square numbers in the table?

Why this symmetry? $1^2=(-1)^2$, $2^2=(-2)^2$, $3^2=(-3)^2$

Modulo 10 multiplication table

x	0	1	2	3	4	5	-4	-3	-2	-1
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	0	2	4	6	8
3	0	3	6	9	2	5	8	1	4	7
4	0	4	8	2	6	0	4	8	2	6
5	0	5	0	5	0	5	0	5	0	5
-4	0	6	2	8	4	0	6	2	8	4
-3	0	7	4	1	8	5	2	9	6	3
-2	0	8	6	4	2	0	8	6	4	2
-1	0	9	8	7	6	5	4	3	2	1

Where are the square numbers in the table?

Why this symmetry? $1^2=(-1)^2$, $2^2=(-2)^2$, $3^2=(-3)^2$, $4^2=(-4)^2$

Abstraction

What is abstraction?

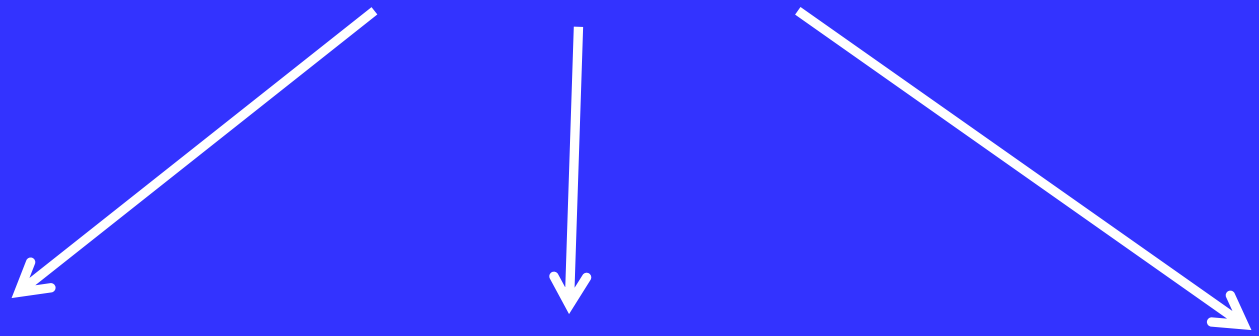
Dog

Abstraction

3

Abstraction

$$a + b = b + a$$



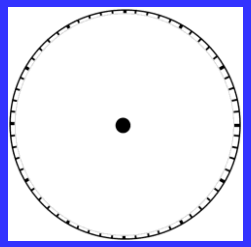
$1 + 2 = 2 + 1$

$3 + 4 = 4 + 3$

$5 + 7 = 7 + 5$

Abstraction

Modulo n



In general, $a \equiv b \pmod{n}$ if $a-b$ is a multiple of n.

Modulo 12

Modulo 4

Modulo 10

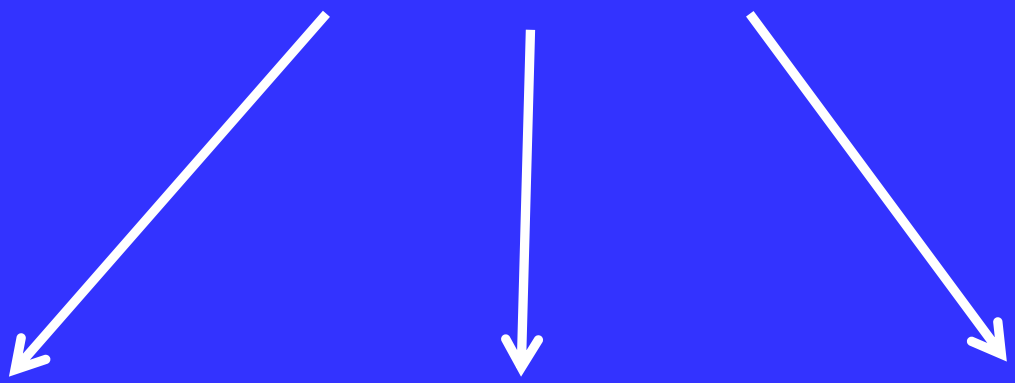
Abstraction

What is Mathematics about?

Mathematics is not only about numbers and figures, it is also about patterns, generalizations and abstractions.

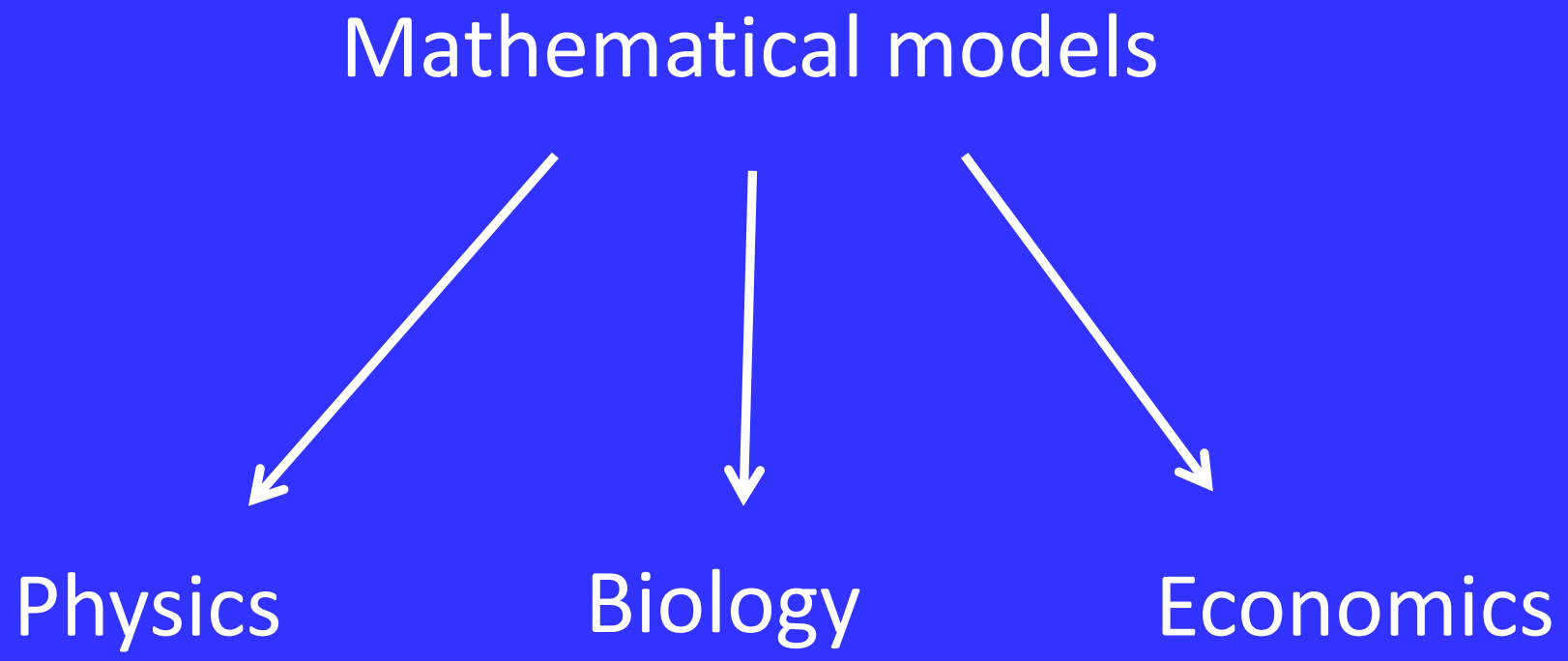
http://en.wikipedia.org/wiki/Mathematics: Mathematics is the abstract study of topics such as quantity (numbers), structure, space, and change.

Abstraction



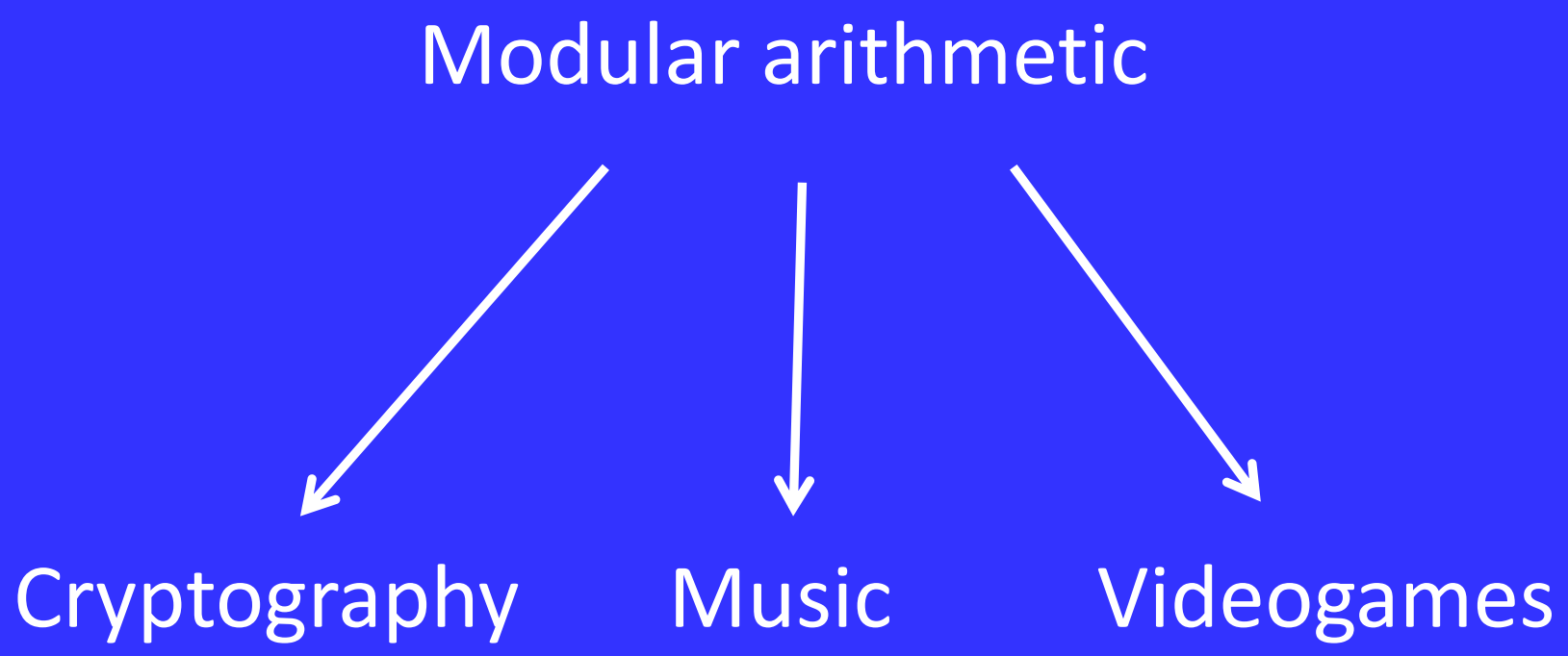
This is an example from Pure Mathematics.

Abstraction



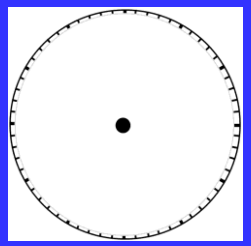
Sometimes more abstract is more useful.

Abstraction



Abstraction helps Unification

Modulo n



Gauss, *Disquisitiones Arithmeticae*, 1801

Modulo 12

Modulo 2

+	Even	Odd
Even	Even	Odd
Odd	Odd	Even

Modulo 10

Last digit arithmetic

Unification in Physics

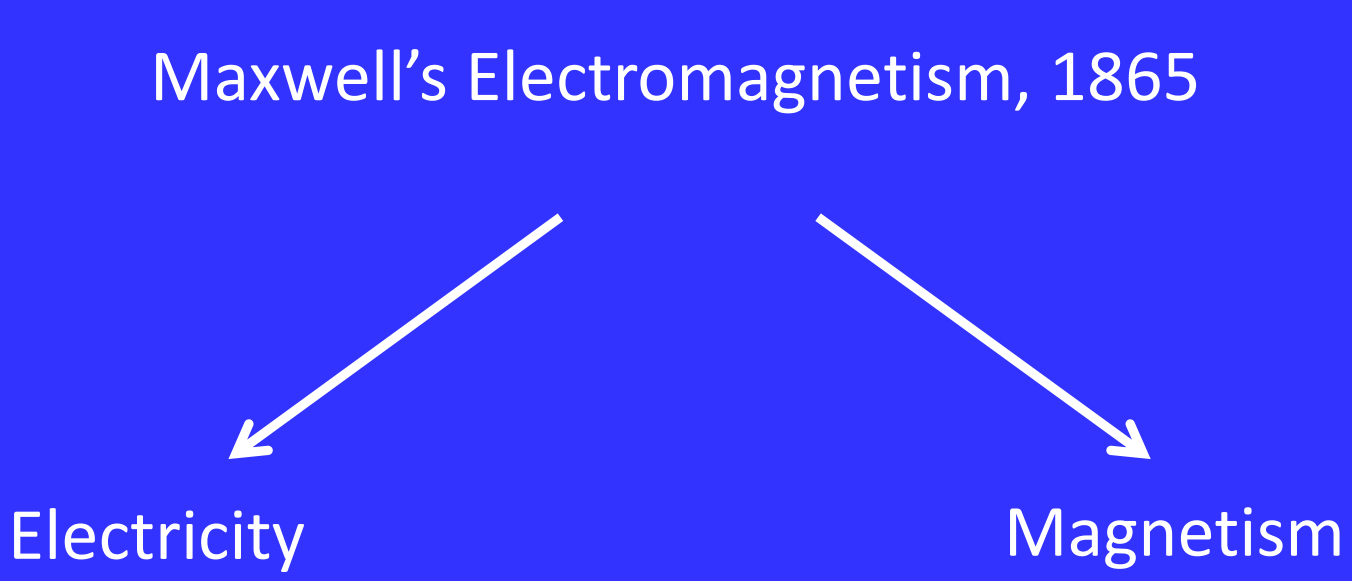
Newton's *Principia*, 1687

Terrestrial Mechanics (Galileo)

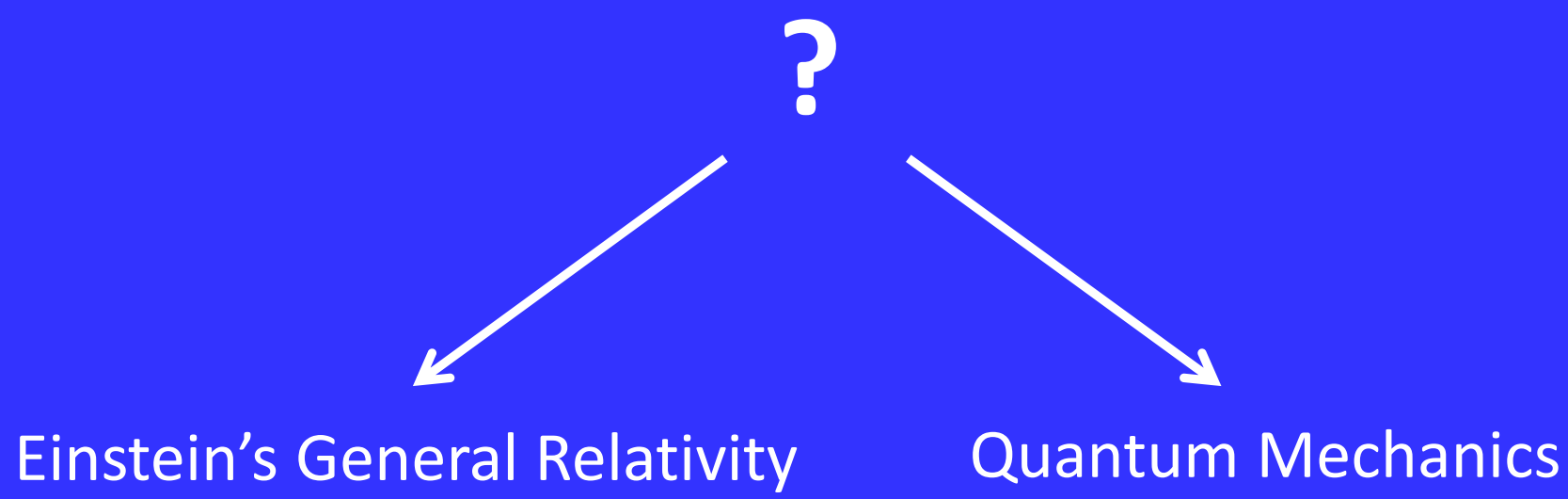
Celestial Mechanics (Kepler)

Thought experiment: Do you know how to see that the Moon is falling just like an apple?

Unification in Physics

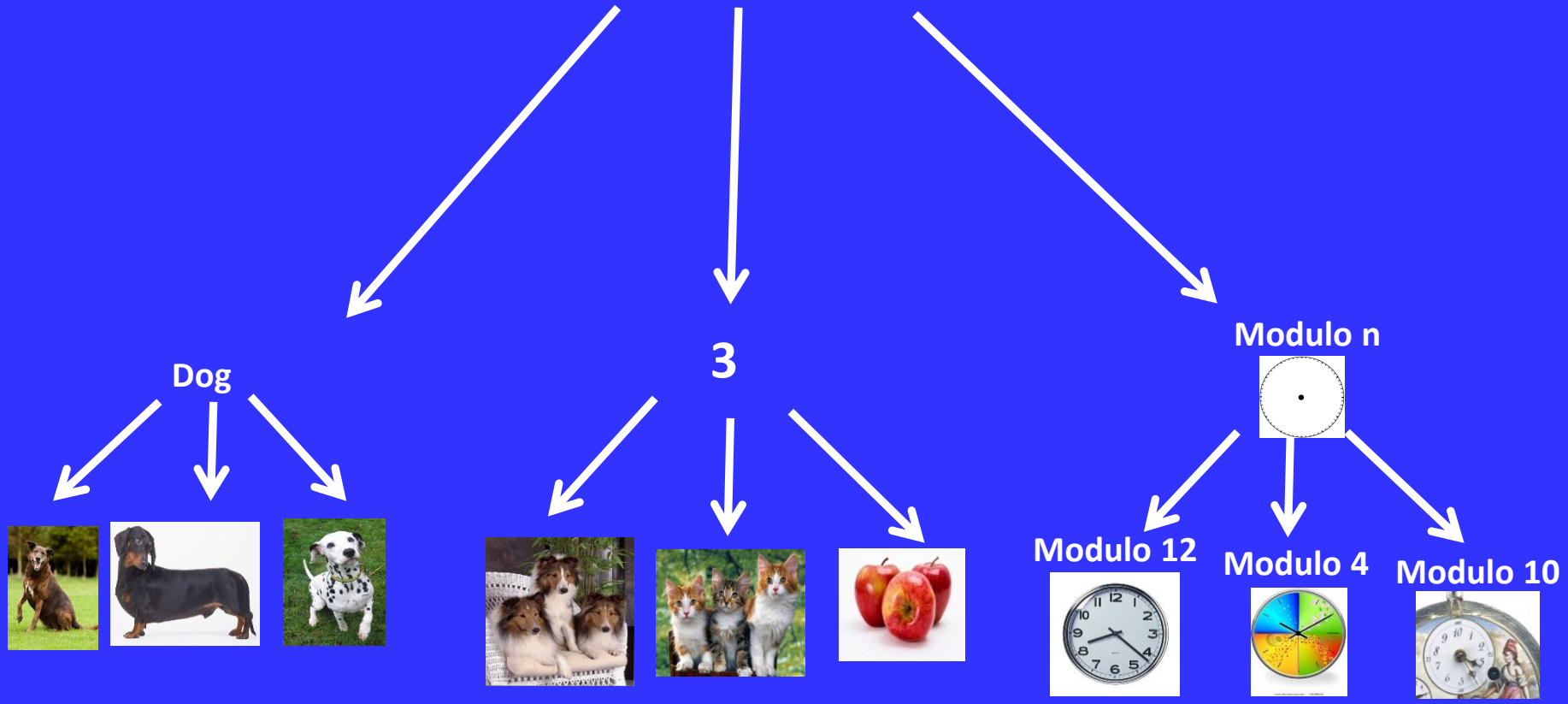


Unification in Physics



Abstraction

Concept of abstraction

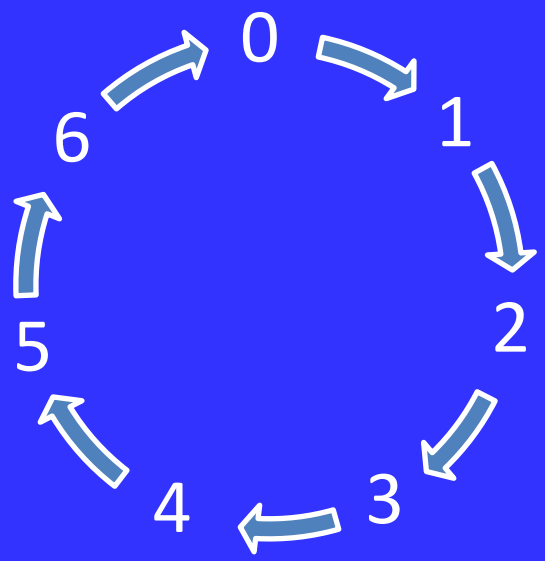


Concepts are important, e.g. chess, programming.

On which day of the week were you born?

What modulo are we going to use?

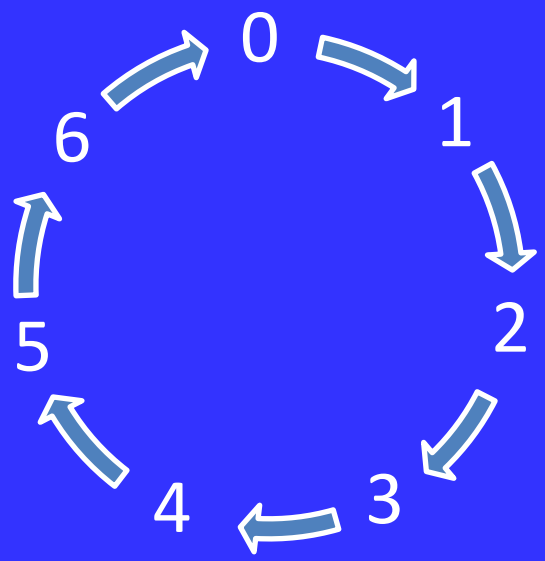
Modulo 7



- 0 represents Sunday
- 2 represents Tuesday
- 4 represents Thursday
- 6 represents Saturday

- 1 represents Monday
- 3 represents Wednesday
- 5 represents Friday

Modulo 7



- 0 denotes 7n: 0, 7, 14, 21,…
- 1 denotes 7n+1: 1, 8, 15, 22,…
- 2 denotes 7n+2: 2, 9, 16, 23,…
- 3 denotes 7n+3: 3, 10, 17, 24,…
- 4 denotes 7n+4: 4, 11, 18, 25,…
- 5 denotes 7n+5: 5, 12, 19, 26,…
- 6 denotes 7n+6: 6, 13, 20, 27,…

Modulo 7 addition table

+	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

Examples:

$1 + 4 \equiv 5 \pmod 7$ $2 + 5 \equiv 0 \pmod 7$ $6 + 2 \equiv 1 \pmod 7$

Monday + 4 = Friday Tuesday + 5 = Sunday Sat + 2 = Monday

Day of the week

- One way to determine the day of the week of a given date is to assign codes to the years, months and dates such that:

  Day of the week ≡ year code
  + month code
  + date (mod 7)

Day of the week

- For simplicity let's choose 1 as the code of the year 2001. We start by noticing that 1/January/2001 was a Monday, so we need:

$$1 \equiv 1 + \text{January code} + 1 \pmod 7$$

Then, what is the January code?

$$0 \equiv \text{January code} + 1 \pmod 7$$

$$\text{January code} \equiv -1 \pmod 7$$

$$\text{January code} \equiv 6 \pmod 7$$

This is the January code for non-leap years.

Month codes

How do we find the February code?

February code ≡ January code

+ number of days in January (mod 7)

February code ≡ 6 + 31 (mod 7)

February code ≡ 37 (mod 7)

February code ≡ 2 (mod 7)

This is the February code for non-leap years.

Please now compute the March to December codes for a non-leap year.

Month codes

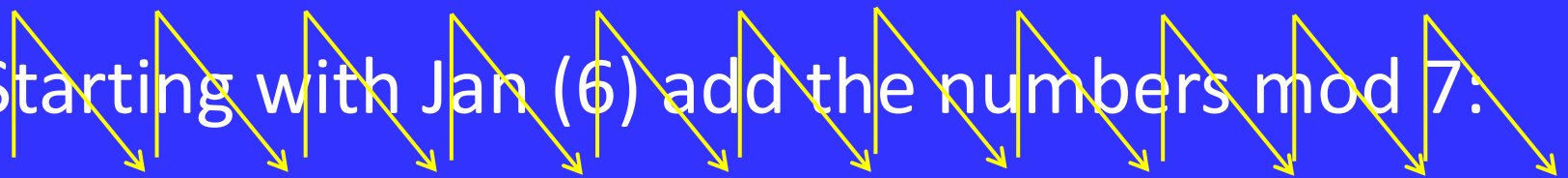
In general, to find the month codes we list the number of days in each month in a non-leap year:

Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec

31 28 31 30 31 30 31 31 30 31 30 31

Look at them modulo 7:

3 0 3 2 3 2 3 3 2 3 2 3



Starting with Jan (6) add the numbers mod 7:

6 2 2 5 0 3 5 1 4 6 2 4

Month codes

How to remember the month codes?

Month	Number	Mnemonic
January	6	WINTER has 6 letters
February	2	February is the 2nd month
March	2	March 2 the beat.
April	5	APRIL has 5 letters
May	0	MAY-0
June	3	Jun (Jun has 3 letters)
July	5	The SHARD (5) opened on July 5
August	1	August begins with A, the 1st letter
September	4	First TERM (4 letters) at school
October	6	SIX or treat!
November	2	11th month (11 => II or 1+1=2)
December	4	LAST (or XMAS) has 4 letters

Month codes

How to remember the month codes?

Remember this number: 622-503-514-624

Or one of these tables:

Jan	6	Feb	2	Mar	2
Apr	5	May	0	Jun	3
Jul	5	Aug	1	Sep	4
Oct	6	Nov	2	Dec	4

May	0
Aug	1
Feb, Mar, Nov	2
Jun	3
Sep, Dec	4
Apr, Jul	5
Jan, Oct	6

Exception: in a leap year the January code is 5 and the February code is 1.

Remember that leap years (almost always) are the years that are multiples of 4.

Year codes

If your birthday fell on a Sunday this year, what day will it fall on next year?

How do we find the 2002 code?

2002 code ≡ 2001 code

+ number of days in 2002 (mod 7)

2002 code ≡ 1 + 365 (mod 7)

2002 code ≡ 1 + 1 (mod 7)

2002 code ≡ 2 (mod 7)

Year codes

In general, to find the year codes we list the number of days in each year:

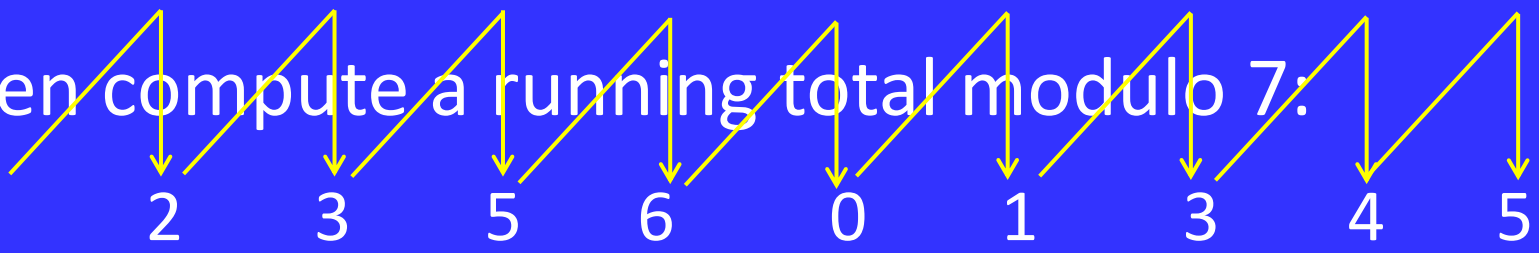
2001 2002 2003 2004 2005 2006 2007 2008 2009 2010

365 365 365 366 365 365 365 366 365 365

Look at them modulo 7:

1 1 1 2 1 1 1 2 1 1

Then compute a running total modulo 7:

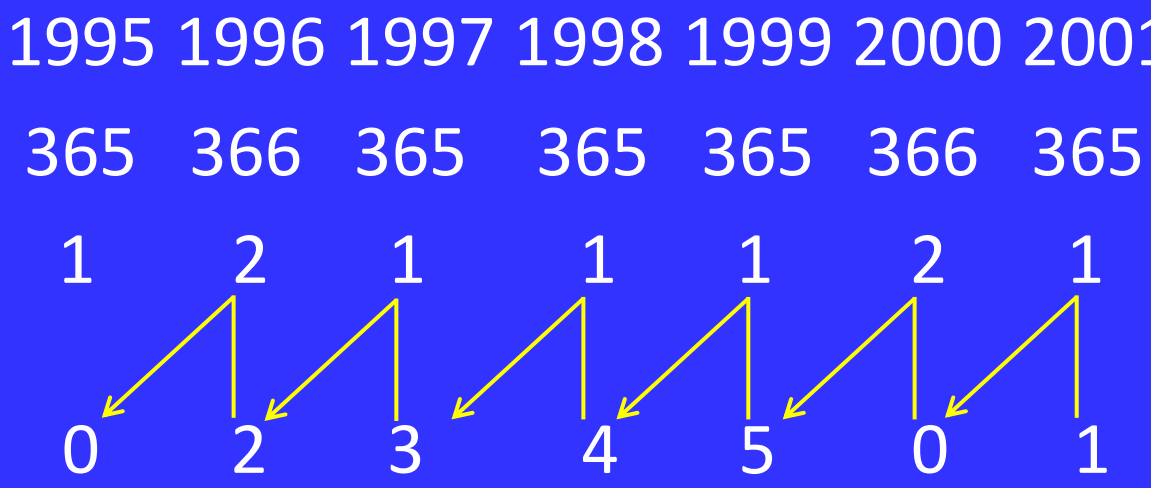


Please now find the 2011-14 codes.

Year codes

What about the 20th century?

- The codes are like the 21st century except that we need to add 1 to the year code. E. g. given that 2001 has code 1, 1901 has code 2.
- We can also deduce codes for previous years:



Year codes

- It helps to remember the years with code 0:

• 1905	Albert Einstein's annus mirabilis
• 1911, 22, 33, 44	First four multiples of 11
• 1916	4^2
• 1939	World War II started
• 1950	Brazil '50 World Cup (Maracanazo)
• 1961	Berlin Wall started
• 1967	The year after 1966!
• 1972	Munich '72 Olympics
• 1978	Argentina '78 World Cup
• 1989	Berlin Wall ended
• 1995	Netscape IPO (Internet boom started)

Year codes

How do we find the 1971 code?

Remember that 1967, the year after England won the World Cup, has code 0.

1971 code ≡ 71-67 + number of leap years in 1968-1971 (mod 7)

1971 code ≡ 4 + 1 (mod 7)

1971 code ≡ 5 (mod 7)

Year codes

- It also helps is to remember years with code equal to its last digit:

  1964-1966

  2000-2003

- Year codes in Excel

- Examples thinking aloud

Worksheet 6
Day of the week

Find the day of the week in which a classmate was born.

Remember:

Day of the week ≡ year code
+ month code
+ date (mod 7)

60 points going around a circle

- Imagine 60 points going around in concentric circles clockwise and starting at 3pm.
- The first, and outermost, point goes around at 1 rpm, the second at 2 rpm and so on until the 60^{th} which goes around at 60 rpm.
- *Where will they be after a minute?*
- *Where will they be after 1/2 minute?*
- *Where will they be after 1/3 minute?*
- *Where will they be after 1/4 minute?*

60 points going around a circle

http://whitneymusicbox.org/whitneyMinute.swf

Pendulum waves

- Something similar can be done with pendulums.
- Each successive shorter pendulum is adjusted so that it executes one additional oscillation in a 1 minute period.
- Video taken from http://www.youtube.com/watch?v=yVkdfJ9PkRQ